Faddeev Equations in the Functional Quantum Theory of the Non-Linear Spinorfield

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Starting with a linear partial differential operator for a certain system of transition amplitudes associated to a state $|a\rangle$ with baryon number $\varrho(a)$ we derive an equation in the $\varrho(a)$-sector of the Functional space which provides the state $|a\rangle$ with an effective potential caused by the polarization cloud. With regard to the needs of n-body scattering we then undertake a cluster decomposition of the effective potential. In particular, we derive the relativistic analoga of Lippmann-Schwinger- and Faddeev equations.

In non-linear spinor theory as treated by Heisenberg [1] and coworkers [2] a physical state $|a\rangle$ is determined by a linear partial differential operator acting in a certain infinite system of transition amplitudes between the vacuum and the state $|a\rangle$. That means, the dynamical behaviour of any asymptotic n-particle configuration is governed by no subsystem other than the whole system of field equations; that is why the Hilbertspace of the field operator cannot have Fock-space structure. On the other hand, the system of field equations carries an algebraic structure which is best represented on Fock-space. Therefore, it is natural to define state vectors by

$$\Phi(j, a) := \sum_{n=0}^{\infty} \int dx_1 \cdots dx_n\, \varphi_n(x_1 \cdots x_1 \,|\, a)$$
$$\cdot j(x_1) \cdots j(x_n)\, \Omega;$$

here the φ's are time-ordered transition amplitudes between the vacuum Ω and state $|a\rangle$, and j is one of the generators of an abstract clifford algebra (j, ϑ, Ω). The set of all state vectors generated by all physical states is called functional state space.

It was Stumpf [3] and coworkers who began constructing the functional state space and therein formulating the essentials of Heisenberg's dipole regularized spinorfield theory. According to Stumpf [4] the Functional Quantum Theory of the non-linear spinorfield has to deal mainly with three problems:

a) derivation of an effective n-particle equation from the fundamental field equation,

b) definition of a physical scalar product in the functional state space,

c) definition of a S-matrix which is unitary.

Ad c): Heisenberg's dipole regularization creates states other than the asymptotic, physical states; therefore the sets of in- and outgoing state vectors need not be equal. But then the S-matrix cannot be unitary when defined as usual. For the problem of unitarization see Stumpf and Scheerer [5].

Ad b): The algebraic definition of a functional state vector makes it evident that the definition of a scalar product in functional state space hinges on the solution theory of the fundamental system of field equations; especially it is not clear what sort of integral has to be used in the definition of a functional state vector. Guided by the experience with the Lee-model one circumvents such difficult questions and forms scalarproducts with the dipole regularized two-point function which is assumed to be compatible with the fundamental dynamical equation, Stumpf [6].

Ad a): To any state $|a\rangle$ with an asymptotic n-particle configuration there is associated a baryon number $\varrho(a)$ which can be used to determine the first φ_n which need not vanish necessarily. Therefore, to the state $|a\rangle$ corresponds a lowest non-trivial sector of the functional state space, the $\varrho(a)$-sector, and it is desirable to reduce the dynamics of the state $|a\rangle$ to the $\varrho(a)$-sector. Such a procedure

has been proposed by Stumpf [8]; his method results in effective potentials for any given n-particle configuration which expresses the influence of higher sectors, called polarization cloud. The effective potentials serve as relativistic analoga of interaction hamiltonians, and contain in principle arbitrary high n-point interactions. In order to test the effective potentials it is necessary to establish the relativistic analoga for Lippmann-Schwinger- and Faddeev-equations and higher sector equations.

In the first section we describe the reduction procedure; then we decompose that potential in the $\varrho(a)$-sector with respect to a cluster representation of the state $|a\rangle$. For $\varrho(a) = 2, 3$ we finally derive our analoga of Lippmann-Schwinger- and Faddeev-equations.

1. The Reduction of the Field Equation

The dynamical field equation in functional Φ-states reads [7]:

$$D_\alpha(x)\,\Phi(j, a) := [\gamma^\mu\,\delta_\mu(x)\,d_\beta(x) + V_\alpha^{\beta\gamma\delta}\,d_\beta(x)\,d_\gamma(x)\,d_\delta(x)]\,\Phi(j, a) = 0\,.$$

Here $d_\beta(x)$ stands for $\vartheta_\beta(x) + F_{\beta\lambda} * j_\lambda(x)$; ϑ and j are the functional destruction- and creation operators, and F is a regularized two-point function. $\Phi(j, a)$ represents the functional construction of a physical state $|a\rangle$:

$$\Phi(j, a) := \sum_{n=0}^\infty \Phi^{(n)}(j, a) := \sum_{n=0}^\infty \int dx_1 \cdots dx_n\, \varphi_n\begin{pmatrix} x_1 \cdots x_n \\ \alpha_1 \quad \alpha_n \end{pmatrix} a\Big)\, j_{\alpha_1}(x_1)\cdots j_{\alpha_n}(x_n)\,\Omega\,.$$

The φ_n's are special transformed expressions of time-ordered transition amplitudes between vacuum Ω and the state $|a\rangle$.

Now we have the following component equations of the field equation for a state $|a\rangle$ with baryon number ϱ [8]:

$$[D_\alpha(x)\,\Phi(j, a)]^{(\varrho-1)} = A_1\Phi^{(\varrho)}(j, a) + A_2\Phi^{(\varrho+2)}(j, a) = 0\,,$$
$$[D_\alpha(x)\,\Phi(j, a)]^{(\varrho+1)} = A_1\Phi^{(\varrho+2)}(j, a) + A_2\Phi^{(\varrho+4)}(j, a) + A_3\Phi^{(\varrho)}(j, a) = 0\,,$$

and for all $4 \leq \lambda \in 2\,\mathbb{N}$:

$$[D_\alpha(x)\,\Phi(j, a)]^{(\varrho-1+\lambda)} = A_1\Phi^{(\varrho+\lambda)}(j, a) + A_2\Phi^{(\varrho-2+\lambda)}(j, \alpha) + A_3\Phi^{(\varrho+2+\lambda)}(j, a)$$
$$+ A_4\Phi^{(\varrho-4+\lambda)}(j, a) = 0$$

with

$$A_1 := \gamma_{\alpha\beta}^\mu\,\delta_\mu(x)\,\vartheta_\beta(x) + 3\,V_\alpha^{\beta\gamma\delta}\,F_{\beta\lambda} * j_\lambda(x)\,\vartheta_\gamma(x)\,\vartheta_\delta(x)\,,$$
$$A_2 := V_\alpha^{\beta\gamma\delta}\,\vartheta_\beta(x)\,\vartheta_\gamma(x)\,\vartheta_\delta(x)\,,$$
$$A_3 := \gamma_{\alpha\beta}^\mu\,\delta_\mu(x)\,F_{\beta\lambda} * j_\lambda(x) + 3\,V_\alpha^{\beta\gamma\delta}\,F_{\beta\lambda} * j_\lambda(x)\,F_{\gamma\nu} * j_\nu(x)\,\vartheta_\delta(x)\,,$$
$$A_4 := V_\alpha^{\beta\gamma\delta}\,F_{\beta\lambda} * j_\lambda(x)\,F_{\gamma\nu} * j_\nu(x)\,F_{\delta\mu} * j_\mu(x)\,.$$

Defining the polarization cloud by $\chi := \sum_{\lambda=0,2}^\infty \Phi^{(\varrho-2+\lambda)}(j, a)$ we have a further equivalent system of equations:

$$\begin{aligned} &[A_1\Phi^{(\varrho)} + A_3\chi]^{(\varrho-1)} = 0 \\ &D_\alpha(x)\,\chi + \{D_\alpha(x) - A_3\}\,\Phi^{(\varrho)} = 0 \end{aligned} \qquad \Leftrightarrow D_\alpha(x)\,\Phi(j, a) = 0\,. \tag{1}$$

Therefore, we start the reduction procedure by generalizing Cramer's rule. Set

$$D_\alpha(x)\,\chi = -\{D_\alpha(x) - A_3\}\,\Phi^{(\varrho)} =: b_\alpha(x)\,, \quad d_\alpha(x)\,\chi =: K_\alpha(x)$$

and $\int dx'\, G_{\alpha\alpha'}(x, x')\,\gamma_{\alpha'\beta}^\mu\,\delta_\mu(x') = \delta_{\alpha\beta}(x)$ and create an inhomogeneous problem in proving the equivalence:

$$\begin{aligned} D_\alpha(x)\,\chi = b_\alpha(x) &\Leftrightarrow K_\alpha(x) + \int dx'\, G_{\alpha\alpha'}(x, x')\,V_{\alpha'}^{\beta\gamma\delta}\,d_\beta(x')\,d_\gamma(x')\,K_\delta(x') \\ &= \int dx'\, G_{\alpha\alpha'}(x')\,b_{\alpha'}(x') =: (Gb)_\alpha(x)\,. \end{aligned}$$

We then see that

$$T_{\alpha\delta}(x, x') := G_{\alpha\alpha'}(x, x')\, V_{\alpha'}^{\beta\gamma\delta}\, d_\beta(x')\, d_\gamma(x')$$

can be treated as the kernel of an integral operator $T(x)$. As a solution of $K(x) + T(x) K(x) = (Gb)(x)$ we take the Neumann expansion

$$K_\delta(x) = (Gb)_\delta(x) + \int \mathrm{d}x'\, T_{\delta\eta}(x, x')\, (Gb)_\eta(x') + \iint \mathrm{d}x'\, \mathrm{d}x''\, T_{\delta\eta}(x, x')\, T_{\eta\mu}(x', x'')\, (Gb)_\mu(x'') + \cdots$$

which we insert in the first line of (1); we get in first order approximation:

$$\{\gamma_{\alpha\beta}^\mu\, \delta_\mu(x)\, \vartheta_\beta(x) + 3\, V_\alpha^{\beta\gamma\delta}\, F_{\beta\lambda} * j_\lambda(x)\, \vartheta_\gamma(x)\, \vartheta_\delta(x) + V_\alpha^{\beta\gamma\delta}\, F_{\delta\nu} * j_\nu(x)\, \vartheta_\beta(x)\, \vartheta_\gamma(x)$$
$$+ V_\alpha^{\beta\gamma\delta} \int \mathrm{d}x'\, G_{\delta\lambda}(x, x')\, 3\, V_\lambda^{\beta'\gamma'\delta'}\, F_{\beta'\nu} * j_\nu(x')\, F_{\gamma'\mu} * j_\mu(x')\, \vartheta_{\delta'}(x')\, \vartheta_\beta(x)\, \vartheta_\gamma(x)\}$$
$$\cdot\, \Phi^{(\varrho)}(j, a) = 0 =: O(j, \vartheta)\, \Phi^{(\varrho)}.$$

Now we can interprete $O(j, \vartheta) - A_1$ as the effective first order potential of the polarization cloud seen by the state $|a\rangle$.

2. Second Quantized Cluster Representations

To motivate our approach let us consider the hamiltonian $h_0 + V$ in Fock-space; here V is a two-body interaction hamiltonian. In the n-th sector of Fock-space we get the well-known expression:

$$-\frac{1}{2m} \sum_{i=1}^n \nabla_i + \sum_{i<\gamma}^n V_{ij}.$$

If $\Phi^{(n)} := A_n \Phi$ is an arbitrary antisymmetrized n-particle state vector we can decompose $\Phi^{(n)}$ as follows:

$$\Phi^{(n)} = \frac{1}{M} \sum_{i=1}^M \chi(E_\nu)\, E_\nu^{-1} A_{n, l}\, \Phi.$$

For, let $X_n := \{1, 2, \ldots, n\}$ be a disjoint union of arbitrary subsets $\emptyset \ne X_i \subset X_n$, $1 \le i \le l \le n$. We call $\{X_i : 1 \le i \le l\}$ a cluster partition and each of its elements a cluster. Let $S(n)$ resp. $S(n_i)$ be the symmetric group of X_n resp. X_i; if the number of elements in X_i is n_i, we have $n = \sum_{i=1}^l n_i$, and

$$S(n, l) := \{\sigma \in S(n) \mid \sigma_{|X_i} \in S(n_i)$$
$$\forall 1 \le i \le l\}$$

is a normal subgroup of $S(n)$. Thereby we get:

$$S(n) = \bigcup_{i=1}^M S(n, l)\, E_i, \quad E_1 = 1,$$
$$E_{i\ne1} \notin S(n, l)$$

with $M = n! / |S(n, l)|$, $|S(n, l)|$ the order of $S(n, l)$. Clearly, the union is disjoint, and having fixed the E_i's it then follows:

$$A_n = \frac{1}{n!} \sum_{\sigma \in S(n)} \chi(\sigma)\, \sigma = \frac{1}{n!} \sum_{\sigma \in \bigcup_{i=1}^M S(n, l) E_i} \chi(\sigma)\, \sigma$$

$$= \frac{1}{M} \sum_{i=1}^M \left(\sum_{\sigma \in S(n, l)} \chi(\sigma)\, \sigma \right) \chi(E_i)\, E_i$$
$$= \frac{1}{M} \sum_{i=1}^M \chi(E_i)\, A_{n, l} E_i:$$

here $A_{n, l}$ is the antisymmetrizer of $S(n, l)$.

Now it is obvious that the decomposition of the state vector $\Phi^{(n)}$ as a sum of different permutations of a partially antisymmetrized state vector $\Phi_{n, l}$ induces a decomposition of the hamiltonian as a sum of different permutations of the hamiltonian restricted to the set of partially antisymmetrized state vectors. But the restricted hamiltonian is the sum of the hamiltonians of the clusters and of the cluster interaction hamiltonians [9]. As for example the Faddeev-equations essentially need the definition of cluster operators, the usefulness of the state vector decomposition is exhibited.

However, if one is working with second quantized hamiltonians and wishes to specify the number of clusters only, but not the n-particle sector, then it is preferable to decompose the clifford algebra (j, ϑ, Ω) [8]. Next we give a representation of such an algebraic structure and show its equivalence to decomposing state vectors.

Let $F_A^{(i)}$, $1 \le i \le l$, be antisymmetric Fock-spaces, $\psi_i^\pm(f_i)$ the associated creation resp. destruction operators, the f_i's belonging to the one-particle subspaces $\mathscr{H}_i$ of $F_A^{(i)} := F_A(\mathscr{H}_i)$.

We then form $\bigotimes_{i=1}^l F_A^{(i)}$ and define

$$F^\pm(f_i): \bigotimes_{i=1}^l F_A^{(i)} \to \bigotimes_{i=1}^l F_A^{(i)} \quad \text{by}$$
$$F_i^\pm(f_i) := 1 \otimes 1 \otimes \cdots \otimes 1 \otimes \psi_i^\pm(f_i) 1 \otimes 1 \otimes \cdots 1.$$

For

$$\bigotimes_{i=1}^{l} F_{A}^{(i)} \ni \Phi := \bigotimes_{i=1}^{l} \Phi_i, \quad \Phi_i \in F_{A}^{(i)},$$

we have in case

$$i \neq j: \ [F_i{}^{\pm}(f_i), \ F_j{}^{\pm}(g_j)]_- \Phi = 0$$

and in case

$$i = j: \ [F_i{}^{\pm}(f_i), \ F_i{}^{\pm}(g_j)]_+ \Phi$$
$$= \bigotimes_{\nu=1}^{i-1} \Phi_\nu \otimes [\psi_i{}^{\pm}(f_i), \psi_j{}^{\pm}(g_j)]_+ \Phi_i \otimes \bigotimes_{\nu=i+1}^{l} \Phi_\nu.$$

Now the quadratic forms

$$H_k := \int dx\, dx'\, V_k(x, x')\, \psi_k{}^{+}(x)\, \psi_k{}^{+}(x')$$
$$\cdot \psi_k{}^{-}(x')\, \psi_k{}^{-}(x)$$

on $F_{A}^{(k)}$, $1 \le k \le l$, give rise to the definitions ($h_0 := (H_k)_{k=1}^{l}$):

$$d\Gamma_l(h_0) = H_1 \otimes 1 \otimes \cdots \otimes 1$$
$$+ 1 \otimes H_2 \otimes 1 \otimes \cdots \otimes 1 + \cdots$$
$$+ 1 \otimes 1 \otimes \cdots \otimes H_l,$$

$$\sum_{k<m}^{l} V_{km} := \sum_{k<m}^{l} \int dx\, dx'\, V_{km}(x, x')$$
$$\cdot F_k{}^{+}(x)\, F_k{}^{-}(x)\, F_m{}^{+}(x')\, F_m{}^{-}(x'),$$

where

$$V_k \in \mathscr{H}_k \otimes_s \mathscr{H}_k, \quad V_{km} \in \mathscr{H}_k \otimes \mathscr{H}_m.$$

We set $H := d\Gamma_l(h_0) + \sum_{k<m}^{l} V_{km}$.

If $n = \sum_{k=1}^{l} n_k$ we let Φ^{n_k} be the vector representing particles in k-particle Fock-space, i.e.

$$\Phi^{n_k} := (0, 0, \ldots, \Phi^{(n_k)}, 0, \ldots) \in F_{A}^{(k)},$$

and we define

$$\Phi^{[n_k]} := \bigotimes_{k=1}^{l} \Phi^{n_k}.$$

Defining the mapping J by

$$J \Phi^{[n_k]} := \bigotimes_{k=1}^{l} \Phi^{(n_k)}$$

we see that J can be extended to a linear isometry of $F_{A}^{[l]} := \sum F_{A}^{[n_k]}$ onto

$$F_{A}^{(l)} := \sum \bigotimes_{k=1}^{l} F_{A}^{(n_k)};$$

here $F_{A}^{[n_k]}$ is the closed subspace

$$\{\Phi^{[n_k]} \mid n = \sum_{k=1}^{l} n_k \text{ fixed}\} \quad \text{in} \quad \bigotimes_{k=1}^{l} F_{A}^{(k)},$$

and $\sum$ means the closed linear hull of the union of the $F_{A}^{[n_k]}$ where the union runs over all $n = \sum_{k=0}^{l} n_k$ with $n \in \mathbb{N} \cup \{0\}$. We remark that $F_{A}^{[l]}$ is dense in $\bigotimes_{k=1}^{l} F_{A}^{(k)}$, therefore J can be extended to $\bigotimes_{k=1}^{l} F_{A}^{(k)}$. Further that $\sum$ is a direct sum.

Now we have for any partition $n = \sum_{k=1}^{l} n_k$:

$$J H \Phi^{[n_k]} = \sum_{k=1}^{l} \Phi^{(n_1)} \otimes \cdots \otimes (H_k \Phi^{n_k})^{(n_k)} \otimes \cdots \otimes \Phi^{(n_k)} + \sum_{k<m}^{l} \int dx\, dx'\, V_{km}(x, x')\, \Phi^{(n_1)}$$
$$\otimes \cdots \otimes (\psi_k{}^{+}(x)\, \psi_k{}^{-}(x)\, \Phi^{n_k})^{(n_k)} \otimes \cdots \otimes (\psi_m{}^{+}(x')\, \psi_m{}^{-}(x')\, \Phi^{n_m})^{(n_m)} \otimes \cdots \otimes \Phi^{(n_l)}$$
$$= \sum_{k=1}^{l} \Phi^{(n_1)} \otimes \cdots \otimes \left(\sum_{i<j}^{n_k} V_{ij}^{(k)} \right) \Phi^{(n_k)} \otimes \cdots \otimes \Phi^{(n_l)}$$
$$+ \sum_{k<m}^{l} \sum_{i}^{n_k} \sum_{j}^{n_m} V_{ij}^{(km)} \Phi^{(n_1)} \otimes \cdots \otimes \Phi^{(n_k)} \otimes \cdots \otimes \Phi^{(n_m)} \otimes \cdots \otimes \Phi^{(n_l)}$$

which is the familiar result we would get by specifying a partition in the n-th sector of classical Fock-space. We set

$$J H \Phi = J H J^{-1} J \Phi =: h J \Phi,$$

where $\Phi \in F_{A}^{[l]}$; clearly h has a cluster representation specifying effective clusterpotentials and effective selfenergy potentials; the algebraic structure is

$$h = \sum \left\{ \left(\sum_{i<j}^{n_k} V_{ij}^{(k)} \right) + \sum_{k<m}^{l} \sum_{i}^{n_k} \sum_{j}^{n_m} V_{ij}^{(km)} \right\},$$

where $\sum$ runs again over all partitions and all $n \in \mathbb{N}$. It remains the task of antisymmetrization. As

$$\bigotimes_{k=1}^{l} F_k^{(n_k)} = \left(\bigotimes_{k=1}^{l} A_{n_k} \right) \left(\bigotimes_{k=1}^{l} F^{(n_{k'})} \right) =: A_{n,l} F^{(n)}$$

we define a mapping $C: \bigotimes_{k=1}^{l} F_{A}^{(n_k)} \to F_{A}^{(n)}$ by

$$A_n = \frac{1}{M} \sum_{\nu=1}^{M} \chi(E_\nu)\, E_\nu{}^{-1} A_{n,l} =: C A_{n,l}.$$

C can again be extended linearly to $F_A^{(l)}$, and C is a partial isometry (note: $F_A^{(n)} \subseteq \overset{l}{\underset{k=1}{\otimes}} F_A^{(n_k)}$). Now we have for any

$$\Phi \in F_A^{[l]}: \quad CJH\Phi = ChJ\Phi = hCJ\Phi$$
$$=: h\Phi_l.$$

For special $\Phi = \Phi^{[n_k]}$ we clearly get:

$$CJH\Phi^{[n_k]}(x_1, \cdots, x_n)$$
$$= \sum_{k<m}^{n} V(x_k, x_m)\Phi^{(n)}(x_1, \cdots, x_n),$$

if the two-body potentials are equal. Let us note that by now we have antisymmetrized $\overset{l}{\underset{k=1}{\otimes}} F_A^{(k)}$, for $CJF_A^{[l]}$ is a totally antisymmetrized subspace in $F_A^{(l)}$ which is isomorphic via J^{-1} to a totally antisymmetrized subspace in $F_A^{[l]}$; as $F_A^{[l]}$ is dense in $\overset{l}{\underset{k=1}{\otimes}} F_A^{(k)}$ the closure of $J^{-1}CJF_A^{[l]}$ is indeed the antisymmetrized tensorproduct of the

$$F_A^{(k)}: \quad \mathrm{cl}\,(J^{-1}CJF_A^{[l]}) = \overset{l}{\underset{k=1}{\otimes}}_A F_A^{(k)}.$$

It follows: $[J, C]_- = 0$.

Because of the above coincidence we think that our definition of a cluster hamiltonian on

$$\overset{l}{\underset{k=1}{\otimes}} F_A^{(k)} \quad \text{by} \quad H = \mathrm{d}\Gamma_l(h_0) + \sum_{k<m}^{l} V_{km}$$

is a useful concept when investigating the structure of cluster dynamics, as all work can be done independently of a specified cluster configuration and particle number. Forinstance, the Faddeev formalism can be formulated on $\overset{l}{\underset{k=1}{\otimes}} F_A^{(k)}$.

On the other side, when doing practical calculations, by the existence of the mapping CJ all operator expressions in $\overset{l}{\underset{k=1}{\otimes}} F_A^{(k)}$ can be transformed in the well-known Fock space expressions. We remark that it is useless to define $\sum_{i=1}^{l} F_i^{\pm}$ and then an interaction hamiltonian as a bilinear form in that superposition; clearly the reason is that CJ is not invertible.

Because of the polarization cloud Functional Quantum Theory is dealing with effects of no finite variable structure; therefore, the generalization to tensor products of the Fock spaces is always unavoidable.

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3. The Relativistic Analoga of Lippmann-Schwinger- and Faddeev-Equations

As our relativistic analogon of the hamiltonian for the two/three-body problem we determine that part in $K(j, \vartheta) := \int \mathrm{d}x \,_\mu \gamma_{\alpha\alpha'} \, \delta^\mu(x) \, j_\alpha(x) \, O(j, \vartheta)$ which contains only up to two pairs of j's and ϑ's. We get:

$$K_2(j, \vartheta) := \int \mathrm{d}x \, \Box(x) \, j_\alpha(x) \, \vartheta_\alpha(x) + \int \mathrm{d}x \,_\mu \gamma_{\alpha\alpha'} \, \delta^\mu(x) \, 4 \, V_{\alpha'}^{\beta\gamma\delta} \, j_\alpha(x) \, F_{\beta\lambda} * j_\lambda(x) \, \vartheta_\gamma(x) \, \vartheta_\delta(x)$$

which is our "$h_0 + V$" in functional space.

The decomposition of K_2 into $\mathrm{d}\Gamma_l(K_2) + \sum_{k<m}^{l} V_{km}$ then reads:

$$\mathrm{d}\Gamma_l(K_2) := \sum_{k=1}^{l} \int \mathrm{d}x \, \Box(x) \, j_\alpha^{(k)}(x) \, \vartheta_\alpha^{(k)}(x) + \sum_{k=1}^{l} \int \mathrm{d}x \,_\mu \gamma_{\alpha\alpha'} \, \delta^\mu(x) \, 4 \, V_{\alpha'}^{\beta\gamma\delta} \, j_\alpha^{(k)}(x) \, F_{\beta\lambda} * j_\lambda^{(k)}(x) \, \vartheta_\gamma^{(k)}(x) \, \vartheta_\delta^{(k)}(x),$$

$$\sum_{k<m}^{l} V_{km} := \sum_{k<m}^{l} \int \mathrm{d}x \,_\mu \gamma_{\alpha\alpha'} \, \delta^\mu(x) \, 4 \, V_{\alpha'}^{\beta\gamma\delta} \, j_\alpha^{(k)}(x) \, \vartheta_\delta^{(k)}(x) \, F_{\beta\lambda} * j_\alpha^{(m)}(x) \, \vartheta_\gamma^{(m)}(x).$$

As in the Heisenberg theory the mass of a meson, for example, can be calculated [10], we use instead of $\mathrm{d}\Gamma_l(K_2)$ the following expression:

$$\mathrm{d}\Gamma_l(K_m) := \sum_{k=1}^{l} \int \mathrm{d}x \, (\Box(x) + m_k^2) \, j_\alpha^{(k)}(x) \, \vartheta_\alpha^{(k)}(x)$$

which is an algebraic decomposition of the selfenergy operator in lowest approximation.

The case $l = 2$.

For a two-particle state $|a\rangle$ we get for $C \circ J \circ K_2$ in the second sector of functional space [9]:

$$\int dx_1 dx_2 [\Box_1 + m_1{}^2 + \Box_2 + m_2{}^2]\, \varphi\begin{pmatrix} x_1\, x_2 \\ \alpha_1\, \alpha_2 \end{pmatrix}\Big|a\Big) D\begin{pmatrix} x_1\, x_2 \\ \alpha_1\, \alpha_2 \end{pmatrix} = - \int dx\, dx'\, \mu \gamma_{\alpha\alpha'}\, \delta^\mu(x)\, 4\, V_{\alpha'}^{\beta\gamma\delta}$$

$$\cdot F_{\beta l}(x - x')\, \varphi\begin{pmatrix} x\, x \\ \delta\, \gamma \end{pmatrix}\Big|a\Big) D\begin{pmatrix} x\, x' \\ \alpha\, l \end{pmatrix} \Leftrightarrow [\Box_1 + m_1{}^2 + \Box_2 + m_2{}^2]\, \varphi\begin{pmatrix} x_1\, x_2 \\ \alpha_1\, \alpha_2 \end{pmatrix}\Big|a\Big)$$

$$= - \mu \gamma_{\alpha_1 \alpha'}\, \delta^\mu(x_1)\, 4\, V_{\alpha'}^{\beta\gamma\delta}\, F_{\beta \alpha_2}(x_1 - x_2)\, \varphi\begin{pmatrix} x_1\, x_1 \\ \delta\, \gamma \end{pmatrix}\Big|a\Big).$$

Now we Fourier-transform the equation and introduce the variables $x_1' := x_1 - x_2$, $x_2' := x_2$:

$$[-p_1{}^2 + m_1{}^2 - p_2{}^2 + m_2{}^2]\, \hat\varphi\begin{pmatrix} p_1\, p_2 \\ \alpha_1\, \alpha_2 \end{pmatrix}\Big|a\Big)$$

$$= - \mu \gamma_{\alpha_1 \alpha'}\, i\, p_1{}^\mu\, 4\, V_{\alpha'}^{\beta\gamma\delta} \int dx_1 dx_2 \exp[i(p_1 x_1 + p_2 x_2)]\, F_{\beta\alpha_2}(x_1 - x_2)\, \varphi\begin{pmatrix} x_1\, x_1 \\ \delta\, \gamma \end{pmatrix}\Big|a\Big)$$

$$= - \mu \gamma_{\alpha_1 \alpha'}\, i\, p_1{}^\mu\, 4\, V_{\alpha'}^{\beta\gamma\delta} \int dx_1 dx_2 \exp[i(p_1 + p_2) x_2' + i p_2 x_1']\, F_{\beta\alpha_2}(x_2')\, \varphi\begin{pmatrix} x_2'\, x_l' \\ \delta\, \gamma' \end{pmatrix}\Big|a\Big)$$

$$\Leftrightarrow [\tfrac12(p_s{}^2 - m_s{}^2) + 2(p_r{}^2 - m_r{}^2)]\, \hat{\tilde\varphi}_T\begin{pmatrix} p_r \\ p_s, \alpha_1\, \alpha_2 \end{pmatrix}\Big|a\Big)$$

$$= - \mu \gamma_{\alpha_1 \alpha'}\, i\,(p_s/2 + p_r)^\mu\, 4\, V_{\alpha'}^{\beta\gamma\delta}\, \hat F_{\beta\alpha_2}(p_s/2 - p_r)\, \tilde\varphi_T\begin{pmatrix} 0 \\ p_s, \delta\, \gamma \end{pmatrix}\Big|a\Big) =: C_{\alpha_1\alpha_2}^{\delta\gamma}(p_s, p_r)\, \tilde\varphi_T\begin{pmatrix} 0 \\ p_s, \delta\, \gamma \end{pmatrix}\Big|a\Big).$$

Here we made use of the transformation

$$T:\ \mathbb{R}^8 \to \mathbb{R}^8$$

given by

$$(x_1', x_2') \to (x_s, x_r) := \left(\frac{x_1' + x_2'}{2}, x_1' - x_2'\right)$$

and its adjoint T^* which in matrix representation reads:

$$\begin{pmatrix} \tfrac12 & 1 \\ \tfrac12 & -1 \end{pmatrix}\begin{pmatrix} p_s \\ p_r \end{pmatrix} = \begin{pmatrix} p_1' \\ p_2' \end{pmatrix}.$$

Further we set: $2\, m_r{}^2 := m_1{}^2 + m_2{}^2 - \tfrac12 m_s{}^2$, and

$$\varphi_T(p_s, 0) := \int dx \exp(i\, p_s x)\, \varphi_T(x, 0).$$

Defining the potential W by

$$W\hat\varphi_T\begin{pmatrix} p_r \\ p_s, \alpha_1\, \alpha_2 \end{pmatrix}\Big|a\Big) := C_{\alpha_1\alpha_2}^{\delta\gamma}(p_s, p_r)\, \varphi_T\begin{pmatrix} 0 \\ p_s, \delta\, \gamma \end{pmatrix}\Big|a\Big),$$

the resolvent g_0 by

$$g_0(z) := [z - \tfrac12(\mathrm{Id}_s{}^2 - m_s{}^2) - 2(\mathrm{Id}_r{}^2 - m_r{}^2)]^{-1}$$

and the resolvent g by

$$g(z) := [z - \tfrac12[\mathrm{Id}_s{}^2 - m_s{}^2] - 2[\mathrm{Id}_r{}^2 - m_r{}^2] - W]^{-1}.$$

we establish a Lippmann-Schwinger-type-equation in formal analogy to [11]:

$$\hat\varphi_{\overline{T}}^\pm\begin{pmatrix} p_r \\ p_s, \alpha_1\, \alpha_2 \end{pmatrix}\Big|a\Big) = \hat\Phi_T^i\begin{pmatrix} p_r \\ p_s, \alpha_1\, \alpha_2 \end{pmatrix}\Big|a\Big)$$

$$+ g_0(p_s, p_r\,|\,a)\, W\, \hat\varphi_{\overline{T}}^\pm\begin{pmatrix} p_r \\ p_s, \alpha_1\, \alpha_2 \end{pmatrix}\Big|a\Big).$$

The case $l = 3$.

Similar to the case $l = 2$ we get the following equation for the partially antisymmetrized φ-function φ_1 (that is, in the variables x_2, x^3 φ_1 is antisymmetric):

$$[\Box_1 + m_1{}^2 + \Box_2 + m_2{}^2 + \Box_3 + m_3{}^2]\, \varphi_1\begin{pmatrix} x_1\, x_2\, x_3 \\ \alpha_1\, \alpha_2\, \alpha_3 \end{pmatrix}\Big|a\Big)$$

$$= - \mu \gamma_{\alpha_1 \alpha'}\, \delta^\mu(x_1)\, 4\, V_{\alpha'}^{\beta\gamma\delta}\, F_{\beta\alpha_2}(x_1 - x_2)\, \varphi_1\begin{pmatrix} x_1\, x_1\, x_3 \\ \delta\, \gamma\, \alpha_3 \end{pmatrix}\Big|a\Big)$$

$$= - \mu \gamma_{\alpha_1 \alpha}\, \delta^\mu(x_1)\, 4\, V_{\alpha'}^{\beta\gamma\delta}\, F_{\beta\alpha_3}(x_1 - x_3)\, \varphi_1\begin{pmatrix} x_1\, x_2\, x_1 \\ \delta\, \alpha_2\, \gamma \end{pmatrix}\Big|a\Big)$$

$$= - \mu \gamma_{\alpha_2 \alpha'}\, \delta^\mu(x_1)\, 4\, V_{\alpha'}^{\beta\gamma\delta}\, F_{\beta\alpha_3}(x_2 - x_3)\, \varphi_1\begin{pmatrix} x_1\, x_2\, x_2 \\ \alpha_1\, \delta\, \gamma \end{pmatrix}\Big|a\Big).$$

Notice that the algebraic structure is the same for all the permuted cluster configurations φ_ν of φ_1. Fourier-transformation then gives

$$[\tfrac13(p_s{}^2 - m_s{}^2) + 2(p_r{}^2 - m_r{}^2) + \tfrac32(p_u{}^2 - m_u{}^2)]\, \widehat{\varphi_{1T}}\begin{pmatrix} p_r \quad p \\ p_s, \alpha_2\, \alpha_3'\alpha_1 \end{pmatrix}$$

$$= \mu \gamma_{\alpha_1 \alpha'}\, i\, p_1{}^\mu\, 4\, V_{\alpha'}^{\beta\gamma\delta}\, \hat F_{\beta\alpha_2}(-p_2)\, \tilde\varphi_{3T}\begin{pmatrix} 0 & -p_r - \tfrac12 p_u \\ p_s, \delta\, \gamma, & \alpha_3 \end{pmatrix}$$

$$- \mu \gamma_{\alpha_1 \alpha'} i \, p_1{}^\mu 4 \, V_{\alpha'}^{\beta \gamma \delta} \, \hat{F}_{\beta \alpha_3}(-p_3) \, \tilde{\varphi}_{2\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ \delta \gamma \end{matrix}, \begin{matrix} p_r - \frac{1}{2} p_u \\ \alpha_2 \end{matrix} \right)$$

$$+ \mu \gamma_{\alpha_2 \alpha'} i \, p_2{}^\mu 4 \, V_{\alpha'}^{\beta \gamma \delta} \, \hat{F}_{\beta \alpha_3}(-p_3) \, \tilde{\varphi}_{\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ \delta \gamma \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right)$$

$$=: C_{\alpha_1 \alpha_2}^{\delta \gamma}(p_s, p_r, p_u) \, \tilde{\varphi}_{3\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ \delta \gamma \end{matrix}, \begin{matrix} -p_r - \frac{1}{2} p_u \\ \alpha_3 \end{matrix} \right) + C_{\alpha_1 \alpha_3}^{\delta \gamma}(p_s, p_r, p_u) \, \tilde{\varphi}_{2\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ \delta \gamma \end{matrix}, \begin{matrix} p_r - \frac{1}{2} p_u \\ \alpha_2 \end{matrix} \right)$$

$$+ C_{\alpha_2 \alpha_3}^{\delta \gamma}(p_s, p_r, p_u) \, \tilde{\varphi}_{1\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ \delta \gamma \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right).$$

Notice that

a) $\begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & 0 & 1 \\ \frac{1}{3} & 1 & -\frac{1}{2} \\ \frac{1}{3} & -1 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} p_s \\ p_r \\ p_u \end{pmatrix}, \quad m_1{}^2 + m_2{}^2 + m_3{}^2 = \frac{1}{3} m_s{}^2 + 2 \, m_r{}^2 + \frac{3}{2} m_u{}^2.$

b) $\tilde{\varphi}(\cdot, 0, \cdot) := \int dp \, \hat{\varphi}(\cdot, p, \cdot).$

c) $\varphi_i := \chi(E_i) \, \varphi \circ E_i^{-1} \quad \text{with} \quad E_1 = (123), \quad E_2 = (212), \quad E_3 = (312);$
   $\varphi_1 = A_{3,2} \varphi_1$, and $\varphi := \frac{1}{3} \sum \varphi_i$

is totally antisymmetric. For full details see [9]. Defining the resolvent g_0 by

$$g_0(z) := [z - \frac{1}{3}(\mathrm{Id}_s{}^2 - m_s{}^2) - 2\,(\mathrm{Id}_r{}^2 - m_r{}^2) - \frac{3}{2}(\mathrm{Id}_u{}^2 - m_u{}^2)]^{-1}$$

and the mappings K_α, $1 \leq \alpha \leq 3$, by

$$K_1 \widehat{\varphi_{1\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) := C_{\alpha_2 \alpha_3}^{lm}(p_s, p_r, p_u) \, \tilde{\varphi}_{1\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right),$$

$$K_2 \widehat{\varphi_{1\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) := C_{\alpha_1 \alpha_3}^{lm}(p_s, p_r, p_u) \, \varphi_{2\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} p_r - \frac{1}{2} p_u \\ \alpha_2 \end{matrix} \right),$$

$$K_3 \widehat{\varphi_{1\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) := C_{\alpha_1 \alpha_2}^{lm}(p_s, p_r, p_u) \, \tilde{\varphi}_{3\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} -p_r - \frac{1}{2} p_u \\ \alpha_3 \end{matrix} \right),$$

$$K_0 := [\frac{1}{3}(\mathrm{Id}_s{}^2 - m_s{}^2) + 2\,(\mathrm{Id}_r{}^2 - m_r{}^2) + \frac{3}{2}(\mathrm{Id}_u{}^2 - m_u{}^2)]$$

our equation reads in the functional space analogon of $\overset{2}{\underset{k=1}{\otimes}} F_\mathrm{A}^{(n_k)}$, $n_1 = 1$ and $n_2 = 2$:

$$\left(K_0 - \sum_{i=1}^{3} K_i \right) \widehat{\varphi_{1\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) = 0.$$

Regarding the fact that Fourier-transformation does not depend of whether choosing first the φ_1-equation and then the φ_r-equation we can complete the definition of the K_i's:

$$K_1 \widehat{\varphi_{r\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) := C_{\alpha_2 \alpha_3}^{lm}(p_s, p_r, p_u) \, \tilde{\varphi}_{r\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix} \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right),$$

$$K_2 \widehat{\varphi_{2\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) := C_{\alpha_1 \alpha_3}^{lm}(p_s, p_r, p_u) \, \widehat{(\varphi_2)}_{2\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} p_r - \frac{1}{2} p_u \\ \alpha_2 \end{matrix} \right)$$

$$= - C_{\alpha_1 \alpha_3}^{lm}(p_s, p_r, p_u) \, \tilde{\varphi}_{1\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} p_r - \frac{1}{2} p_u \\ \alpha_2 \end{matrix} \right),$$

$$K_3 \widehat{\varphi_{2\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) := C_{\alpha_1 \alpha_2}^{lm}(p_s, p_r, p_u) \, \widetilde{(\varphi_2)}_{3\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} -p_r - \frac{1}{2} p_u \\ \alpha_3 \end{matrix} \right)$$

$$= - C_{\alpha_1 \alpha_2}^{lm}(p_s, p_r, p_u) \, \tilde{\varphi}_{3\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} -p_r - \frac{1}{2} p_u \\ \alpha_3 \end{matrix} \right),$$

and

$$K_2 \widehat{\varphi_{3\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) := - C_{\alpha_1 \alpha_3}^{lm}(p_s, p_r, p_u) \, \tilde{\varphi}_{1\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} p_r - \frac{1}{2} p_u \\ \alpha_2 \end{matrix} \right),$$

$$K_3 \widehat{\varphi_{3\mathrm{T}}} \left(p_s, \begin{matrix} p_r \\ \alpha_2 \alpha_3 \end{matrix}, \begin{matrix} p_u \\ \alpha_1 \end{matrix} \right) := C_{\alpha_1 \alpha_2}^{lm}(p_s, p_r, p_u) \, \tilde{\varphi}_{3\mathrm{T}} \left(p_s, \begin{matrix} 0 \\ lm \end{matrix}, \begin{matrix} -p_r - \frac{1}{2} p_u \\ \alpha_3 \end{matrix} \right).$$

Because of $\varphi = \frac{1}{3}\sum_{\nu=1}^{3}\varphi_\nu$ in the functional analogon of $F_A^{(3)}$ we have the equation:

$$(K_0 - \sum_{i=1}^{3} K_i)\widehat{\varphi_T} = 0.$$

It is obvious that this equation results from the mapping $C \circ J$.

Finally we discuss the formal deduction procedure of Faddeev-equations [10]. A well-known resolvent equation of the two-body system reads:

$$g = g_0 + g_0 V g = g_0 + g V g_0$$

where g_0 is the free resolvent, g the full resolvent and V the potential. It follows:

$$g = g_0 + g_0[V + VgV]g_0 =: g_0 + g_0 t g_0.$$

t is the t-matrix of two-body scattering. Repeating the iteration we get:

$$g = g_0 + g_0 V g_0 + g_0 V g_0 t g_0,$$

and $t = V + V g_0 t$.

If we set $K := \sum_{i=1}^{3} K_i$ the same relations hold true for the three-body system:

$$T = K + KGK = K + Kg_0 T.$$

It follows that $GK = g_0 T$. Defining

$$T_i := K_i + K_i GK$$

we see that

$$T_i = K_i + K_i g_0 T$$

holds, a relation which is called Faddeev-Ansatz. Now the equivalence holds:

$$T_i = K_i + K_i g_0 T \Leftrightarrow [1 - K_i g_0]^{-1}$$
$$\cdot \{K_i + K_i g_0 \sum_{j=1}^{3}(1 - \delta_{ij})T_j\}.$$

As in the two-body system $t = (1 - V g_0)^{-1} V$ holds, we see that $t_i := [1 - K_i g_0]^{-1} K_i$ is the extension of the matrix t to the three-body system; the extension is best described by the expressions $C_{\alpha\beta}^{lm}$; algebraically they are the same both in two- and three-particle systems, but they differ as to the arguments:

two-particle system: $C^{lm}(p_s, p_r)$,

three-particle system: $C^{lm}(p_s, p_r, p_u)$.

Inserting t_i we get as Faddeev-equation for T:

$$T_i = t_i + \sum_{j=1}^{3} t_i g_0 (1 - \delta_{ij})T_j$$
$$=: t_i + \sum_{j=1}^{3} F_{ij} g_0 T_j.$$

Because of $T = \sum_{i=1}^{3} T_i$ we have further:

$$G = g_0 + \sum_i g_0 T_i g_0 =: g_0 + \sum G_i$$
$$= g_0 + \sum_i \{g_0 t_i g_0 + g_0 \sum_j F_{ij} g_0 T_j g_0\}.$$

As in two-body systems the relation $g - g_0 = g_0 t g_0$ holds, we have $g_i - g_0 = g_0 t_i g_0$ with g_i the full cluster resolvent: $g_i := (z - K_0 + K_i)^{-1}$. So we get:

$$G = g_0 + \sum_j G_i$$
$$= \sum_i \{g_i - g_0 + g_0 \sum_j F_{ij} G_j\} + g_0.$$

Notice that all operators are defined on totally anti-symmetric states because g_0 and K_i are defined there.

We now demand what in the nonrelativistic case can be proved:

$$\lim_{\varepsilon \to 0} \pm i\varepsilon g_i(\pm i\varepsilon)\widehat{\varphi_{\nu T}}$$
$$= \delta_{i\nu} \lim_{\varepsilon \to 0} \pm i\varepsilon g_\nu(\pm i\varepsilon)\widehat{\varphi_{\nu T}} =: \delta_{i\nu}\widehat{\Phi_{\nu T}^{\pm}},$$
$$i = 0, 1, 2, 3.$$

That means there exist free channel functions $\widehat{\Phi_{\nu T}^{\pm}}$ which are in some sense orthogonal. Interpreting all resolvents in the limit $\lim_{\varepsilon \to 0}$, we finally have the Faddeev-equations for the channel functions:

$$\widehat{p_{\nu T}^{\pm}} := G\widehat{\varphi_{\nu T}} = \sum_i G_i \widehat{\varphi_{\nu T}} =: \sum_i (\widehat{\varphi_{\nu T}^{\pm}})^{(i)}$$
$$= \sum_i (g_i - g_0 + g_0 \sum_j F_{ij} G_j)\widehat{\varphi_{\nu T}},$$
$$\Leftrightarrow (\widehat{\varphi_{\nu T}^{\pm}})^{(i)} = \delta_{i\nu}\widehat{\Phi_{\nu T}^{\pm}} + g_0 \sum_j F_{ij}(\widehat{\varphi_{\nu T}^{\pm}})^{(i)},$$
$$\widehat{\varphi_{\nu T}^{\pm}} = \sum_i (\widehat{\varphi_{\nu T}^{\pm}})^{(i)}.$$

Having solved this system of coupled equations the totally antisymmetrized solution reads:

$$\widehat{\varphi_T^{\pm}} := G\widehat{\varphi_T} = \frac{1}{3}\sum_\nu \widehat{\varphi_{\nu T}^{\pm}}.$$

Approximating $(1 - K_i g_0)^{-1}$ by $1 + K_i g_0$ and recalling the definition of the K_i's we get for the first channel function:

$$(\widehat{\varphi_{1T}^{\pm}})^{(1)} = \widehat{\Phi}_{1T}^{\pm} + g_0[1 + K_2 g_0] K_2 (\widehat{\varphi_{1T}^{\pm}})^{(2)} + g_0[1 + K_3 g_0] K_3 (\widehat{\varphi_{1T}^{\pm}})^{(3)},$$

$$(\widehat{\varphi_{1T}^{\pm}})^{(2)} = g_0[1 + K_1 g_0] K_1 (\widehat{\varphi_{1T}^{\pm}})^{(1)} + g_0[1 + K_3 g_0] K_3 (\widehat{\varphi_{1T}^{\pm}})^{(3)},$$

$$(\widehat{\varphi_{1T}^{\pm}})^{(3)} = g_0[1 + K_1 g_0] K_1 (\widehat{\varphi_{1T}^{\pm}})^{(1)} + g_0[1 + K_2 g_0] K_2 (\widehat{\varphi_{1T}^{\pm}})^{(2)}$$

or, in full details:

$$(\widehat{\varphi_{1T}^{\pm}})^{(1)} \left(p_s, {p_r \atop \alpha_2 \alpha_3}, {p_u \atop \alpha_1}\right) = \widehat{\Phi}_{1T}^{\pm} \left(p_s, {p_r \atop \alpha_2 \alpha_3}, {p_u \atop \alpha_1}\right) + g(p_s, p_r, p_u) C_{\alpha_1 \alpha_3}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{2T}^{\pm}})^{(2)} \left(p_s, {0 \atop lm}, {p_r - \frac{1}{2} p_u \atop \alpha_2}\right)$$

$$- \int dp_r' \, g_0(p_s, p_r', p_r - \tfrac{1}{2} p_u) C_{\alpha_3 m}^{hk} (p_s, p_r', p_r - \tfrac{1}{2} p_u) g_0(p_s, p_r, p_u)$$

$$\cdot C_{\alpha_1 \alpha_3}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{1T}^{\pm}})^{(2)} \left(p_s, {0 \atop hk}, {p_r' - \frac{1}{2} p_r + \frac{1}{4} p_u \atop l}\right)$$

$$+ g_0(p_s, p_r, p_u) C_{\alpha_1 \alpha_2}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{3T}^{\pm}})^{(3)} \left(p_s, {0 \atop ln}, {-p_r - \frac{1}{2} p_u \atop \alpha_3}\right)$$

$$+ \int dp_r' \, g_0(p_s, p_r', -p_r - \tfrac{1}{2} p_u) C_{\alpha_3 l}^{hk} (p_s, p_r', -p_r - \tfrac{1}{2} p_u) g_0(p_s, p_r, p_u)$$

$$\cdot C_{\alpha_1 \alpha_2}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{3T}^{\pm}})^{(3)} \left(p_s, {0 \atop hk}, {-p_r' + \frac{1}{2} p_r + \frac{1}{4} p_u \atop m}\right);$$

$$(\widetilde{\varphi_{1T}^{\pm}})^{(2)} \left(p_s, {p_r \atop \alpha_2 \alpha_3}, {p_u \atop \alpha_1}\right) = g_0(p_s, p_r, p_u) C_{\alpha_2 \alpha_3}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{1T}^{\pm}})^{(1)} \left(p_s, {0 \atop lm}, {p_u \atop \alpha_1}\right)$$

$$+ \int dp_r' \, g_0(p_s, p_r', p_u) C_{lm}^{hk} (p_s, p_r', p_u) g_0(p_s, p_r, p_u) C_{\alpha_2 \alpha_3}^{ln} (p_s, p_r, p_u)$$

$$\cdot (\widetilde{\varphi_{1T}^{\pm}})^{(1)} \left(p_s, {0 \atop hk}, {p_u \atop \alpha_1}\right) + g_0(p_s, p_r, p_u) C_{\alpha_1 \alpha_2}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{3T}^{\pm}})^{(3)} \left(p_s, {0 \atop lm}, {-p_r - \frac{1}{2} p_u \atop \alpha_3}\right)$$

$$+ \int dp_r' \, g_0(p_s, p_r', -p_r - \tfrac{1}{2} p_u) C_{\alpha_3 l}^{hk} (p_s, p_r', -p_r - \tfrac{1}{2} p_u) g_0(p_s, p_r, p_u)$$

$$\cdot C_{\alpha_1 \alpha_2}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{3T}^{\pm}})^{(3)} \left(p_s, {0 \atop hk}, {-p_r' + \frac{1}{2} p_r + \frac{1}{4} p_u \atop m}\right);$$

$$(\widehat{\varphi_{1T}^{\pm}})^{(3)} = g_0(p_s, p_r, p_u) C_{\alpha_2 \alpha_3}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{1T}^{\pm}})^{(1)} \left(p_s, {0 \atop lm}, {p_u \atop \alpha_1}\right)$$

$$+ \int dp_r' \, g_0(p_s, p_r', p_u) C_{lm}^{hk} (p_s, p_r', p_u) g_0(p_s, p_r, p_u)$$

$$\cdot C_{\alpha_2 \alpha_3}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{1T}^{\pm}})^{(1)} \left(p_s, {0 \atop hk}, {p_u \atop \alpha_1}\right)$$

$$+ g_0(p_s, p_r, p_u) C_{\alpha_1 \alpha_3}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{2T}^{\pm}})^{(2)} \left(p_s, {0 \atop lm}, {p_r - \frac{1}{2} p_u \atop \alpha_2}\right)$$

$$- \int dp_r' \, g_0(p_s, p_r', p_r - \tfrac{1}{2} p_u) C_{\alpha_2 m}^{hk} (p_s, p_r', p_r - \tfrac{1}{2} p_u) g_0(p_s, p_r, p_u)$$

$$\cdot C_{\alpha_1 \alpha_3}^{lm} (p_s, p_r, p_u) (\widetilde{\varphi_{1T}^{\pm}})^{(2)} \left(p_s, {0 \atop hk}, {p_r' - \frac{1}{2} p_r + \frac{1}{4} p_u \atop l}\right).$$

As next these Faddeev equations will be tested by a pion-nucleon calculation; the Lippmann-Schwinger equation has already been tested by a nucleon-nucleon calculation the results of which will be published in the thesis of Herzog [12].

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